A Continuous Shape Prior for MRF-based Segmentation

Dmitrij Schlesinger

Dresden University of Technology

Motivation



Continuous



- + Low-level properties length, curvature etc.
- + Shape modeling
- Statistical interpretation, learning

Discrete



- Metrication artifacts
- + Spatial relations
- + Multi-label problems
- Locality
- + Sound statistical interpretation

Try to combine ...

Model





x – observation, y – labeling, ϕ – shape function

$$p(x, y; \phi) \propto \exp\left[\alpha \cdot \sum_{rr'} \delta(y_r \neq y_{r'}) + \sum_r y_r \cdot \phi(r)\right] \times \prod_r p(x_r|y_r)$$

The prior consists of two parts – Potts terms and shape terms. Conditionally independent observation model (Gaussian mixtures).



Segmentation:

Let ϕ be known. Bayesian Decision Task with Hamming Loss \rightarrow Maximum Marginal Decision:

$$y_r^* = \operatorname*{arg\,max}_k p(y_r = k | x; \phi)$$

Gibbs Sampling for approximate estimation

Appearances:

Standard EM-scheme:

$$\sum_{r} p(y_r = k | x; \phi) \cdot \ln \sum_{i} w_{ik} \mathcal{N}(x_r; \mu_k, \Sigma_k) \to \max_{w_{ik}, \mu_k, \Sigma_k}$$

Posterior marginal label probabilities are again necessary

Estimation of the shape function



Inference or learning ?

Regularized Likelihood:

$$L(\phi) + \mathcal{R}(\phi) =$$

= $\ln \sum_{y} p(x, y; \phi) - \int_{\Omega} (\lambda_1 \cdot \|\nabla \phi\|^2 + \lambda_2 \cdot \bigtriangleup \phi^2) d\omega \to \max_{\phi}$

The gradient with respect to ϕ :

$$\frac{\partial}{\partial \phi(r)} = p(y_r = 1 | x; \phi) - p(y_r = 1; \phi) + \frac{\partial \mathcal{R}(\phi)}{\partial \phi(r)}$$

- Posterior marginal label probabilities (Gibbs Sampling)
- Prior marginal label probabilities (an approximation)
- Gâteaux derivative (convolution)





Three blocks working in parallel:

- Gibbs Sampling: computes marginal posterior label probabilities
- Appearances: performs the learning of the appearances via EM
- Shape: estimates the shape function via gradient ascent

Experiments



Weizmann Horse Database, 328 images

Good (< 1%) \leftarrow Satisfactory (8 – 9%) \rightarrow Bad (>25%)



Original images



Segmentations (green - false positive, magenta - false negative)



Estimated shape functions (zero-level in red)



Approach	Accuracy (%)	
ObjCut	96.0	
Levin	95.5	
Borenstein	93.6	Complex methods
Zhang	95.4	
Bertelli	94.3	
Domke	92.0	
Our	92.2	
GrabCut	85.5	
Co-segmentation	80.1	Simple methods
MNcut	51.0	

Conclusion





- + Low-level properties length, curvature etc.
- + Shape modeling (possible)
- + Multi-label problems (one shape function per label)
- + Spatial relations (possible, easy)
- + Statistic interpretation (for the discrete part)
- Quite slow (actually, under circumstances)
- Non-convex optimization, initialization is necessary
- Approximate inference

Outlook



Compare

"usual" cont. optimization:
$$\int_{\Omega} \left[\text{Data}(x,\phi) + \text{Regularizer}(\phi) \right] d\omega$$
ours:
$$L(x;\phi) + \int_{\Omega} \text{Regularizer}(\phi) d\omega$$

In both cases the gradient looks like

$$\frac{\partial E(\phi)}{\partial \phi(r)} = \text{Something}(\phi(r)) + \frac{\partial \text{ Regularizer}(\phi)}{\partial \phi(r)}$$

 \Rightarrow other regulizers can be easily adopted \Rightarrow advanced methods from continuous optimization can be used

Multi-label problems \rightarrow part-based shape segmentation